

BRIDGE COURSE

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MATDIP301

Third Semester B.E. Degree Examination, July/August 2022 Advanced Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions.

1. a. Find the modulus and amplitude of the complex number $\frac{4+2i}{2-3i}$. (06 Marks)
- b. Express the complex number $\frac{(1-i)(2-i)}{3-i}$ in the form $x+iy$. (06 Marks)
- c. (i) Define complex number
(ii) If $(3x-2iy)(2+i)=10(1+i)$, then find the values of x and y . (08 Marks)

2. a. Find the n^{th} derivative of e^{ax} . (06 Marks)
- b. Find the n^{th} derivative of $\cos^2 3x$. (06 Marks)
- c. If $y = \tan^{-1} x$, prove that $(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$. (08 Marks)

3. a. Find the angle between the radius vector and the tangent to the curve $r = a(1 - \cos\theta)$. (06 Marks)
- b. Obtain the Maclaurin's series expansion of e^x , upto the term containing x^4 . (06 Marks)
- c. With usual notations, prove that (i) $p = r \sin \phi$ (ii) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4} \left(\frac{dr}{d\theta}\right)^2$. (08 Marks)

4. a. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$, when $f = \log(x^2 + y^2)$. (06 Marks)
- b. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$. (06 Marks)
- c. If $u = x^2 - y^2$, $v = y^2$, find the value of $\frac{\partial(u,v)}{\partial(x,y)}$. (08 Marks)

5. a. Evaluate $\int_{y=0}^{y=1} \int_{x=0}^{x=6} xy \, dx \, dy$. (06 Marks)
- b. Evaluate $\int_0^{\pi} \cos^5\left(\frac{x}{2}\right) dx$. (06 Marks)
- c. Obtain the reduction formula for $\int \sin^n x \, dx$, where n is a positive integer. (08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg, 42+8 = 50, will be treated as malpractice.

- 6 a. Evaluate $\iint_R xy \, dx \, dy$, where R is the region in positive quadrant for which $x + y \leq 1$. (06 Marks)
- b. Evaluate $\int_{z=-1}^1 \int_{y=-2}^2 \int_{x=-3}^3 dx \, dy \, dz$. (06 Marks)
- c. (i) Define Gamma function.
 (ii) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. (08 Marks)
- 7 a. Solve: $\frac{dy}{dx} = \frac{\cos^2 y}{\cos^2 x}$. (06 Marks)
- b. Solve: $(y^3 - 3x^2y)dx + (3xy^2 - x^3)dy = 0$. (06 Marks)
- c. Solve: $\left(y\left(1 + \frac{1}{x}\right) + \cos y\right)dx + (x + \log x - x \sin y)dy = 0$. (08 Marks)
- 8 a. Solve: $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. (06 Marks)
- b. Solve: $(D^2 + 6D + 9)y = 0$. (06 Marks)
- c. Solve: $(D^3 - 7D - 6)y = 0$. (08 Marks)
